AIP ADVANCES **4**, 124307 (2014)

Birefringent phononic structures

I. E. Psarobas,[a] D. A. Exarchos, and T. E. Matikas
Dept. of Materials Science and Engineering, University of Ioannina, 451 10 Ioannina, Greece

(Received 17 October 2014; accepted 9 December 2014; published online 16 December 2014)

Within the framework of elastic anisotropy, caused in a phononic crystal due to low crystallographic symmetry, we adopt a model structure, already introduced in the case of photonic metamaterials, and by analogy, we study the effect of birefringence and acoustical activity in a phononic crystal. In particular, we investigate its low-frequency behavior and comment on the factors which determine chirality by reference to this model. *© 2014 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License.* [http://dx.doi.org/10.1063/1.4904812]

The existence of acoustical activity, analogous to optical activity, has been postulated, long time ago, for liquid crystals.[1] In such a case, as a result of spatial dispersion, there exist two acoustically rotated transverse acoustic waves. Acoustical activity occuring in natural crystals as a first-order spatial dispersion effect has been theoretically studied and presented by Portigal *et al.*[2] In addition, the concept of rotation-gradient theory was also introduced as an essentially different symmetry process observed in acoustically active crystals.[3] Acoustical activity is usually encountered in systems which exhibit spectral nonreciprocity [$\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$],[3] an in principle lack of space-inversion and/or time-reversal symmetries. In this context, the occurrence of topologically nontrivial phenomena, such as phononic chiral edge states, in appropriately designed phononic structures,[4] which would be nonreciprocal by design, one can realize reflection-free one-way transport devices for vibrations,[5,6] to say the least. A low-symmetry phononic structure will definitely introduce a sufficient elastic anisotropy which will eventually lead to what is known as acoustic birefringence.[7]

In our investigation we employ the layer-multiple-scattering (LMS) method,[8] which is well documented for the elastodynamics of phononic crystals of spherical[9] and nonspherical particles.[10] The method, based on an ab initio multiple scattering theory,[11] constitutes a powerful tool for an accurate description of the elastic (acoustic) response of composite structures comprised of a number of different layers having the same 2D periodicity in the $x-y$ plane (parallel to the layers). An advantage of the method is that it does not require periodicity in the z direction (perpendicular to the layers).

A procedure originally suggested by Karathanos *et al.*,[12] regarding the behavior of artificial anisotropic and chiral photonic structures, will be adopted here for studying the phononic response of structures mimicking the same patterns of anisotropy and analogous locally resonant behavior.[13] The goal of such an attempt is to propose low-frequency 3D acoustic diodes,[5] as well as acoustic metamaterials functioning as chiral seismic attenuators.[14] 3D acoustic diodes can be realized in the sense of a passive design, where acoustic isolation is achieved by means of unidirectional (one-way) phononic band gaps spanning over wide regions of the spectrum. Such designs resemble the cases examined here, but with higher asymmetry (e.g. a monoclinic crystal) and size variation of the spheres inside the unit cell.

A birefringent phononic crystal can be conceived as a structure that exhibits elastic anisotropy, so with analogy optical birefringent crystals, one expects transverse elastic waves of opposite

[a]Also at Section of Solid State Physics, The University of Athens, Panepistimioupolis, 157 84 Athens, Greece; Electronic mail: ipsarob@phys.uoa.gr

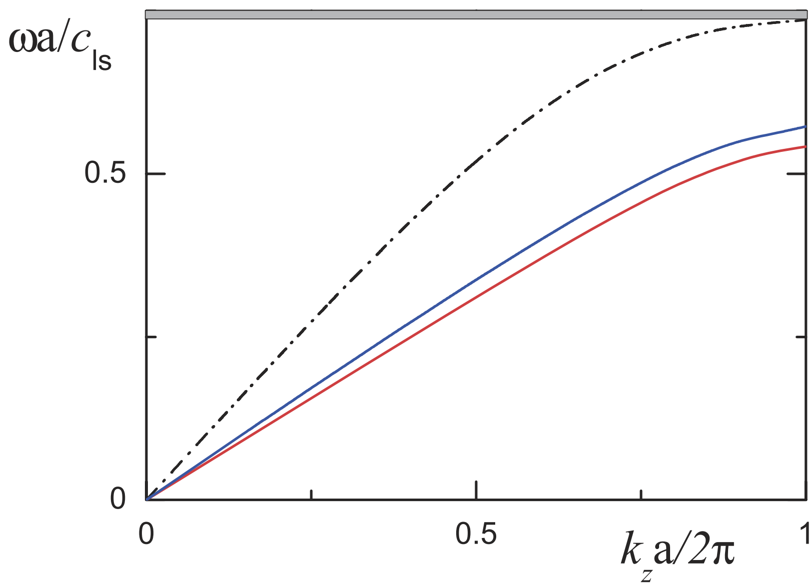

FIG. 1. Non-degenerate shear phonon states (solid curves) of an orthorombic phononic crystal at normal view ($\mathbf{k}_{\parallel} = \mathbf{0}$). The dotted curve corresponds to the compressional band, whereas the gray top is the lower part of the phononic gap of the system. For steel we have used $\rho_s = 7.8$g/cm^3, c_{ls} = 5940 m/s and c_{ts} = 3200 m/s. For epoxy $\rho_x = 1.19$g/cm^3, c_{lx} = 2860 m/s and c_{tx} = 1800 m/s.

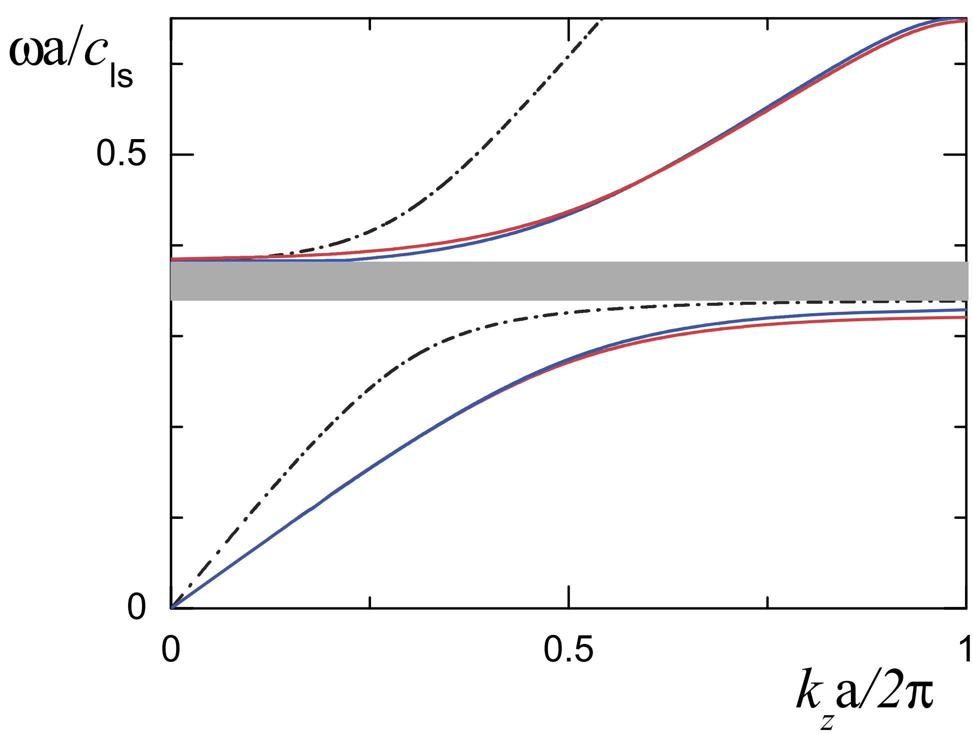

FIG. 2. Normal view ($\mathbf{k}_{\parallel} = \mathbf{0}$) of the frequency band structure of a locally resonant birefrigent phononic crystal. The crystal possesses the same symmetry as in Fig. 1. The spheres have a steel core radius S_{core} = 0.30a and a rubber coating S_{coat} = 0.8a thick. For the rubber we have used $\rho_r = 1.13$g/cm^3, c_{lr} = 1400 m/s and c_{ts} = 100 m/s. Non-degenerate shear phonon states correspond to solid curves and dotted curves correspond to compressional states.

handedness to propagate with different speeds.[15] Such a structure can be realized by an orthorombic crystal of steel spheres in an epoxy matrix as a succession of crystallographic planes of the same 2D periodicity defined by the primitive vectors $\mathbf{a}_1 = (\mathrm{a}, 0, 0)$ and $\mathbf{a}_2 = (0, m\mathrm{a}, 0)$ [namely the (001) crystallographic surface]. Such an assembly of planes of spheres parallel to the $x - y$ plane with the primitive translation along the z-direction, defined by $\mathbf{a}_3 = (0, 0, n\mathrm{a})$, form a 3D crystal of obviously low crystallographic symmetry. For this specific example, we have chosen $m = 1.8$, $n = 1.2$ and a sphere radius $S = 0.38\mathrm{a}$. In order to calculate the complex frequency band structure of the above crystal associated with the elastic field in the manner described in Ref. 9, we impose periodic boundary conditions and for a given angular frequency ω and reduced wave vector $\mathbf{k}_{\parallel}$, we obtain the eigenmodes of the elastic field by determining k_z. The reduced wave vector $\mathbf{k}_{\parallel}$ parallel to the crystallographic plane of stacking and ω are given conserved quantities. k_z follows from the definition of the wave vector $\mathbf{k} = [\mathbf{k}_{\parallel}, k_z(\omega, \mathbf{k}_{\parallel})]$ of a generalized Bloch wave. At low frequencies, the amplitude and polarization of the shear elastic field associated with the $\mathbf{g} = \mathbf{0}$ component of the Block wave ($\mathbf{g}$ are the 2D reciprocal lattice vectors) better describe the phononic bands obtained in Fig. 1. Over a region of frequencies just below the first phononic gap, two shear bands, $k_z^{(x)}(\omega; \mathbf{k}_{\parallel} = \mathbf{0})$ and $k_z^{(y)}(\omega; \mathbf{k}_{\parallel} = \mathbf{0})$, appear. The asymmetry introduced with respect to the x and y directions manifests itself with two distinct bands, which in an otherwise symmetric structure would appear as one doubly degenerate state. The physical interest with respect to elastic anisotropy can be described by $\Delta k_z(\omega)$, being the difference of the wave vector between the two states. For normal incidence ($\mathbf{k}_{\parallel} = 0$) on a slab of the crystal, a wave will accumulate a phase shift as a result of having two different phase velocities inside the crystal equal to

$$\phi(\omega) = \Delta k_z(\omega) d \, , \tag{1}$$

where d denotes the thickness of the slab. Thus, depending on the relative magnitude of the x and y components of the incident field as well as the thickness of the slab, the transmitted shear wave will be linearly, circularly, or elliptically polarized.

Locally resonant behavior of the same system can be achieved by introducing a rubber coating to the spheres and a hybridization gap[8] will reveal in the low frequency regime. This is depicted in

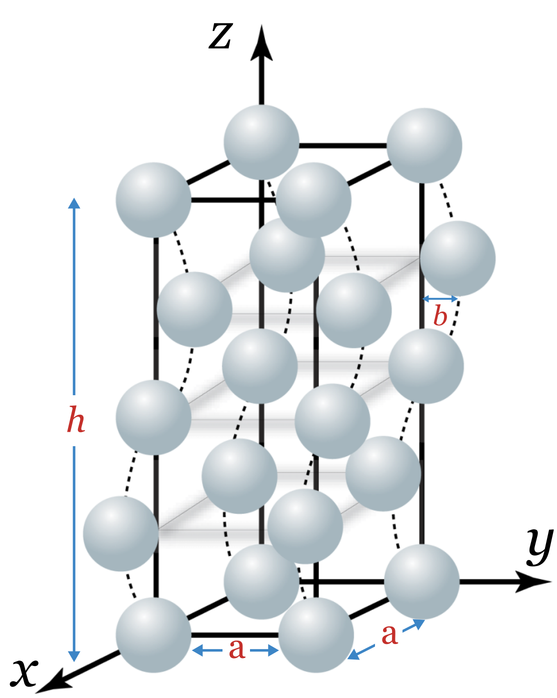

FIG. 3. An acoustically active phononic crystal.

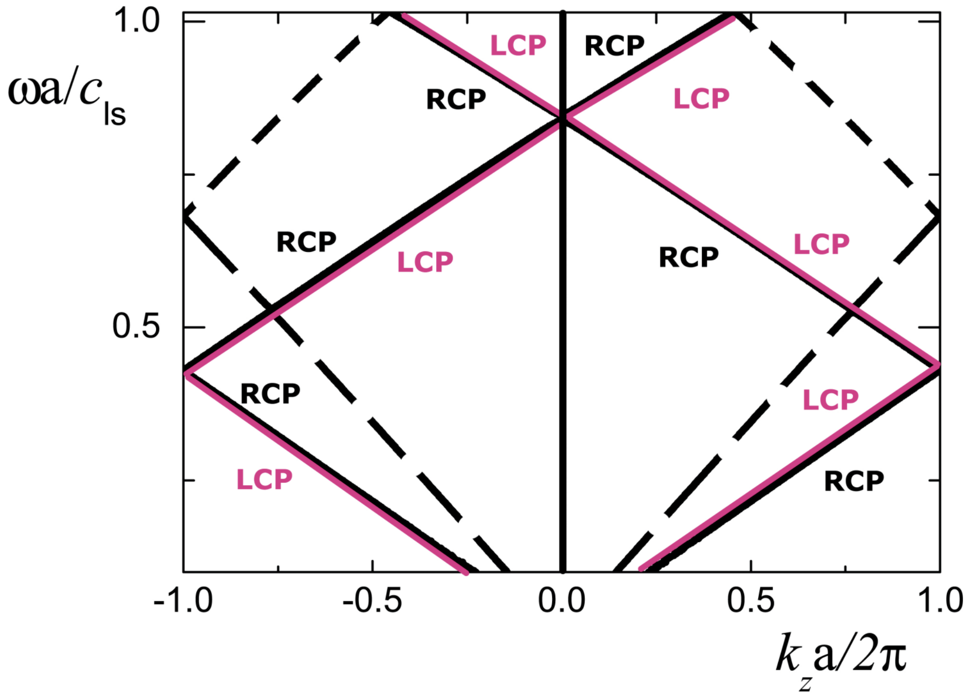

FIG. 4. Normal view ($\mathbf{k}_{\|} = \mathbf{0}$) of the frequency band structure of a chiral phononic crystal. The crystal is made of steel spheres in epoxy, exactly as presented in Fig. 1. The dashed curve corresponds to compressional states, whereas the solid curves are LCP and RCP shear states. Chirality is defined by $h/\mathrm{a} = 2$, $b/\mathrm{a} = 0.25$, and $S/\mathrm{a} = 0.2$.

Fig. 2, where more sophisticated shear phonon states appear. Conceptually, one can optimize such systems to operate as low frequency sensors, where polarization dependance is a critical physical parameter. On the other hand one can design multi-component low-frequency sensitive filters and/or attenuators.

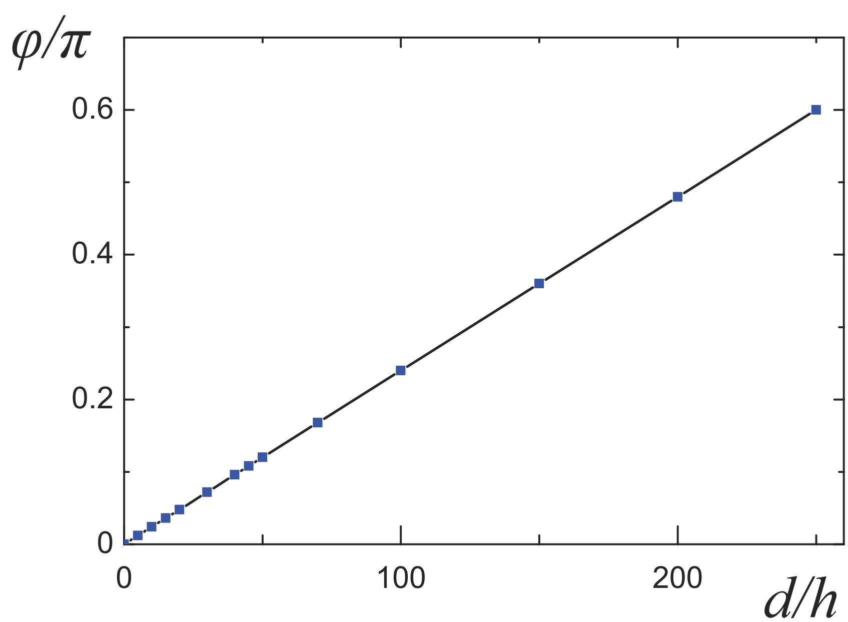

FIG. 5. Low frequency rotatory power of the plane of polarization with the thickness of the slab, when $\omega \mathrm{a}/c_{ls} \simeq 0.3$. The parameters of the chiral structure are the same as in Fig. 4.

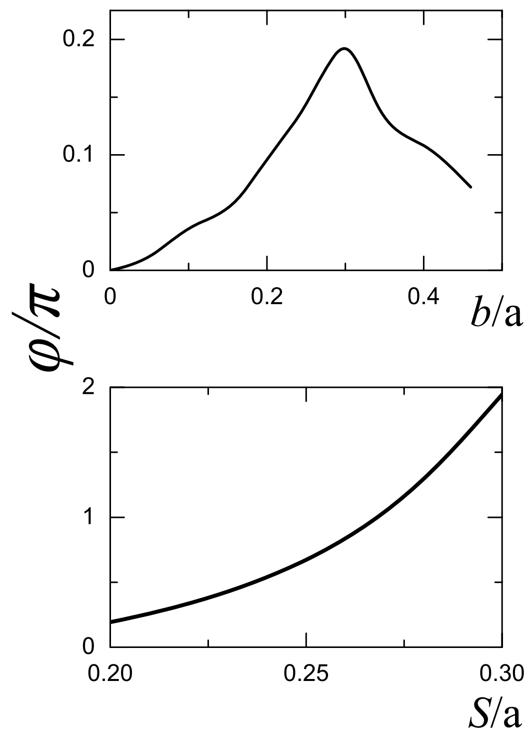

FIG. 6. Low frequency rotatory power of the plane of polarization of a slab 120 layers thick. The frequency is $\omega a/c_{ls} \simeq 0.3$. The upper diagram corresponds to $h/a = 2$ and $S/a = 0.2$, and the lower to $h/a = 2$ and $b/a = 0.3$.

Chiral media circularly polarize the transverse components of a vector field so that propagation occurs in the form of left-circularly polarized (LCP) and right-circularly polarized (RCP) waves, which have different phase speeds. Acoustic chirality, also known as acoustical activity, has been observed experimentally in crystalline matter,[16,17] and it may well serve as a guiding principle in our investigation. Spiral geometry can definitely achieve artificial chiral structures[18] and by analogy to artificial optical activity,[12] one can actually envision acoustically active phononic materials. Such a crystal is shown in Fig. 3 and can be described as a tetragonal crystal of steel spheres in epoxy with a four-point basis. Each plane of spheres, parallel to $x-y$ plane, has the same 2D periodicity, and form a tetragonal 2D lattice defined by the primitive vectors

$$\mathbf{a}_1 = (\mathrm{a}, 0, 0)\ ,\ \mathbf{a}_2 = (0, \mathrm{a}, 0). \tag{2}$$

The four non-primitive planes of spheres defined by the four-point basis, centered at $(0,0,0)$, $(b, 0, h/4)$, $(b, b, h/2)$, and $(0, b, 3h/4)$, define a unit layer. Two successive unit layers along the z direction of the crystal are separated by a primitive translation

$$\mathbf{a}_3 = (0, 0, h)\ . \tag{3}$$

Spheres spiraling along the z axis is indicated by the dotted lines in Fig. 3. The crystal is viewed along the z direction and the spheres are all the same. The field components associated with $\mathbf{g} = \mathbf{0}$ form, in the frequency spectrum, two distinct shear bands associated with the $k_z^{(+)}$ states

$$\mathbf{u}_0^{(+)} = (\hat{\mathbf{x}} + i\hat{\mathbf{y}})u_0^{\pm}\ , \tag{4}$$

corresponding to LCP waves (positive helicity), whereas for the $k_z^{(-)}$ states

$$\mathbf{u}_0^{(-)} = (\hat{\mathbf{x}} - i\hat{\mathbf{y}})u_0^{\pm}\ , \tag{5}$$

we have RCP waves (negative helicity). Fig. 4 shows the frequency band structure of the chiral phononic crystal and clearly demonstrates the polarization behavior of the shear wave states, as defined above.

Explicit computations of a transmitted wave at selected frequencies[9] prove that the rotatory power is proportional to the number d/h of unit layers in a slab of the crystal. The variation of the angle of rotation is demonstrated in Fig. 5.

Finally, from Fig. 6 we can extract some useful information regarding the geometry aspects of a chiral phononic crystal and how they affect its response. It should also be mentioned that the higher the elastic contrast parameters of the spheres and the host materials, the stronger the effect of acoustical activity is demonstrated by the crystal.

We have investigated cases of birefringent phononic structures, which can be realized by employing low-symmetry phononic crystals. Such anisotropy comes from the fact that the crystallographic axes of the crystal are non-equivalent. We studied the low-frequency behavior of such a material and have shown that locally resonant phononic crystals of such behavior can further enhance polarization sensitivity and serve as elastic isolators. Birefringent phononic materials when used in conjunction with losses and disorder can actually provide a scheme for designing next-generation low-frequency filters and shields for vibrations. We have further investigated the special case of chirality with a model phononic structure that mimics spiral geometry. Such an artificial structure exhibits acoustical activity, which rotates the plane of polarization for shear elastic waves. Assemblies of artificial chiral phononic structures can serve as acoustic diodes and with losses, they can be used as seismic attenuators.

This research has been co-financed by the European Union (European Regional Development Fund- ERDF) and Greek national funds through the Operational Program Education and Lifelong Learning 2007-2013 of the National Strategic Reference Framework (NSRF 2007-2013), Action ARISTEIA II.

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